1 Questions & Solutions

1. Before the discovery of the neutron, the nucleus was pictured as consisting of $A$ protons and $A-Z$ electrons. Discuss arguments against this hypothesis. (2 points)

There are a number of arguments that oppose the presence of electrons in nuclei: 1) The uncertainty principle would predict that the momentum of the electrons is $p_c > \frac{\hbar c}{R}$ or larger than about 100 MeV. Such energetic electrons are not emitted by nuclei. 2) An atom with an odd number of electrons would require a nucleus with an odd charge. For instance, $^{14}\text{N}$, would have to have 14 protons to get its mass and 7 electrons to obtain the correct charge of $7e$. This odd number of total particles predicts that such a nucleus is a Fermion and have a half integer spin. However, the ground state of $^{14}\text{N}$ has spin 1.

2. Does a particle with zero electric charge necessarily have no interaction with an external magnetic field? Give an example of a neutral particle that does interact with an electromagnetic field. Find an example for a particle that does not. Does a particle with electric charge necessarily interact with an external magnetic field? (2 points)

A particle with zero charge can have a charge distribution and therefore can interact with an external electromagnetic field. A neutral particle may also have a magnetic moment. The neutron is an example. A neutral particle that has no electromagnetic interaction is the neutrino. A charged particle necessarily interacts with an electromagnetic field.

3. Based on the masses of the heavy gauge boson ($W^\pm, Z$), what is the range of the weak force? Compare to the force in nuclei. (2 points)

The range for the $W$ is $\frac{\hbar c}{m_W c^2} = 0.197 \text{ GeV fm}/81 \text{ GeV} = 2.4 \times 10^{-16} \text{ cm}$. For the mass of the $Z^0$, $m_Z = 91 \text{ GeV}/c^2$ we find $\frac{\hbar c}{m_Z c^2} = 2.2 \times 10^{-16}$.

4. An electron of 100 MeV energy strikes a lead nucleus.

(a) Compute the maximum possible momentum transfer and the recoil energy given to the lead nucleus under these conditions.

(b) Show that the electron can be treated as a massless particle for this problem

(2 points)
a) The maximum momentum transfer occurs when the electron is scattered backwards. To a very good approximation the recoil of the lead target can be neglected and $p_e = -p'_e$. So the maximum momentum transfer is $2p_e = 200\text{MeV}/c$. The recoil energy is $p'^2/2M = 0.1\text{MeV}$.

b) For a massless particle $E_e = p_e c$. For a non-zero mass: $p_e c = \sqrt{E^2_e - m^2_e c^4} \approx E_e - m^2_e c^4/2E_e$. If $E_e = 100\text{MeV}$ then, $p_e c = 100 - (0.511)^2/200 = 99.9987\text{MeV}$. This is sufficiently close to $100\text{MeV}$ to justify the approximation.

5. Consider the following spherical charge distribution:
   \[ \rho(x) = \rho_0 \text{ for } x \leq R \]
   \[ \rho(x) = 0 \text{ for } x > R \]
   (a) Compute the form factor for this uniform charge distribution
   (b) Calculate the mean square radius $\langle x^2 \rangle$
   (3 points)
   
   For any spherical distribution
   \[ F(q^2) = 4\pi\hbar \int x^2 \rho(x) \frac{\sin(qx/\hbar)}{qx} dx \] (1)
   
   For the given distribution, the density $\rho(x)$ is $\rho_0$ for $x < R$ and 0 for $x > R$. Thus,
   \[ F(q^2) = 4\pi\hbar\rho_0 \frac{R}{q} \int_0^R x \sin(qx/\hbar) dx \] (2)
   
   Integration gives
   \[ F(q^2) = \frac{3[\sin(qR/\hbar) - (qR/\hbar)\cos(qR/\hbar)]}{(qR/\hbar)^3} \] (3)
   
   The mean square radius $\langle x^2 \rangle$ is
   \[ \langle x^2 \rangle = \int x^2 \rho_0 d^3x = 4\pi\rho_0 \int x^4 dx = 4\pi\rho R^5/5 \] (4)

6. Describe what muonic atoms are and why they are used for studying the nuclear structure? (2 points)
Muonic atoms are those in which an inner electron is replaced by a muon. Because the muon is about 200 times more massive than an electron it gets 200 times closer to the nucleus and in an s-state can effectively probe the nuclear charge distribution.

7. On conservation laws:
   (a) Assume fermion number conservation but not separate lepton and baryon number conservation. List some possible decay modes of a proton into a lepton and other particles. What is the minimum number of particles required? Why?
   (b) List some decays of the proton that do not conserve $B$ and $L$ separately but conserve $B+L$. Repeat for $B-L$.
   (c) Same as a) but for decays into antileptons plus other particles.
Examples are $p \rightarrow \pi^+ \nu_e$, $p \rightarrow e^- \pi^+ \pi^+$.

The minimum number of other particles required is one in order to conserve energy and momentum.

For $B+L$ conserved, $p \rightarrow e^+ \nu_e \nu_e$, $p \rightarrow \bar{\nu}_e \nu_e \nu_e \pi^+$.

The minimum number of other particles required is now two in order to conserve Fermion number.

8. What does the generalized Pauli principle state? (1 point)

9. Justify that the isospin of the deuteron is zero by either using experimental information or by considering the generalized Pauli principle. (2 points)

10. The reaction $dd \rightarrow \alpha \pi^0$ has been observed but with a very small cross-section. (See E.J. Stephenson et al., Phys. Rev. Lett. 91, 142302, 2003) What does the abnormally small cross-section of the reaction tell us? (2 points)

The absence tells us that isospin is conserved since isospin on the left side of the reaction is zero and on the right side is 1. Or it states that the isospin of the pion is not zero. Indeed, we know that the isospin of the pion is 1.

11. What isospin value would you expect for the ground state of an odd-mass nuclide $(Z,N)$ in the single-particle shell model? (1 point)

12. Assume the nucleus $^{170}$Hf to be a rigid body. You can assume a sphere. Calculate approximately the centrifugal force in the state $J=20$. What would happen to the nucleus if its mechanical properties were similar to those of steel? Support your conclusions with a crude calculation. (2 points)

For a solid sphere $I = \frac{2}{5}AmR_0^2$. $I \approx 5 \times 10^{-54}$ kg m$^2$. The centrifugal force is given by: $F = L^2/(I R_0) = J(J+1) \frac{\hbar^2}{(I R_0)} = 140$ N. If the mechanical properties were like steel the nucleus would fly apart. The binding forces for steel are ionic or electrical and the distances are atomic. Assume that the binding force $f \approx e^2/r^2$ with $r \approx 10^{-8}$ cm. Then we have: $f \approx 10^{-8}$ N. This is much smaller than the centrifugal force.

13. Consider a nucleus with $A=237$. Use the semiempirical mass formula to find $Z$ for the most stable isobar. (2 points)

For a fixed $A$, evaluate $dB/dZ$. The most stables nucleus occurs when $dB/dZ = 0$ and we find: $Z=91$.

14. The binding energy $B$ that is required to disintegrate a nucleus into its constituents neutrons and protons is given by $B = (Zm_p + Nm_n - m_{\text{nuclear}}(Z,N))c^2$ where $m_{\text{nuclear}}(Z,N)$ is the mass of a nucleus with with $Z$ protons and $N$ neutrons. The average binding energy per nucleon in a nucleus is about 8 MeV. Estimate the magnitude of the correction that must be applied to this equation to take into account atomic binding effects. (1 point)

The binding energy of an atom can be approximated by the ionization energy. The last electron is shielded from the nucleus by all the other electrons. We can use the hydrogen atom to estimate the ionization energy of an atom. For hydrogen the binding energy of the
electron is 13.5 eV. This is also approximately the ionization energy of atoms and is much less than the average binding energy per nucleon in a nucleus of about 8 MeV. The correction is therefore of the order of $10^{-5}$.

15. The cross-section for the absorption of antineutrinos with energies as emitted by nuclear reactors is about $10^{-43}$ cm$^2$.

(a) Compute the thickness of a water absorber needed to reduce the intensity of an antineutrino beam by a factor of 2.

(b) Consider a liquid scintillator with a volume of $10^8$ liters and an antineutrino beam with an intensity of $10^{13}$ $\nu_e$/cm$^2$sec. How many capture events are expected a day?

(c) How can the antineutrino capture be distinguished from other reactions?

(3 points)

The absorption length is $1/\sigma n$ where $n$ is the number of target particles per unit volume. We find the absorption length is $2.99 \times 10^{20}$ cm. The intensity is $I = I \exp(-x/\text{absorption length})$. Thus $x = 2.07 \times 10^{15}$ km.

The number of captures/sec is $N = \text{flux} \times \text{total number} \times \text{cross-section}$. With a water-like substance we find $\sim 2890$ captures/day.

The capture reaction on protons is $\nu_e p \rightarrow e^+ n$. A positron and neutron are formed. The signal is distinct from neutrino elastic scattering, for example, $\nu_e + e \rightarrow \nu_e + e$.

16. The decay rate, the inverse of the lifetime, is proportional to the square of the strength of an interaction. The typical electromagnetic decay lifetime is $10^{-18}$ sec.

(a) Compute the electrostatic interaction between two protons inside a nucleus.

(b) Estimate the lifetime of a proton if it decayed through the gravitational force.

(c) What is the current best limit on the proton lifetime, and where does it come from?

(3 points)

The electrostatic interaction strength is $e^2/r = (e^2/hc)(hc/r) = (1/137) (197.3 \text{ MeV fm} / 1 \text{ fm}) = 1.4 \text{ MeV}$. The strength of the gravitational interaction between two protons at a distance of 1fm is $Gm^2/r = 1.2 \times 10^{-36}$ MeV. The ratio of the gravitational to electrostatic potential is about $10^{-36}$. Since a typical electromagnetic decay lifetime is $10^{-18}$ s, the gravitational lifetime would be $(10^{36})^2 \times 10^{-18} = 3 \times 10^{46}$ yr.