Lecture 11
- Neutrinos and Neutrino Oscillations -

Experimental Nuclear Physics PHYS 741

heeger@wisc.edu

References and Figures from:
- Basdevant et al., “Fundamentals in Nuclear Physics”
- Henley et al., “Subatomic Physics”
- Oser, “Lake Luise Lectures”
# Lecture Schedule

Note: lectures highlighted in red need to be rescheduled, lectures in blue have been rescheduled.

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<th>Lecture</th>
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<td>ch10 of McKeown, nu osc articles</td>
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<td>13</td>
<td>radiation and matter</td>
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Course Projects

Malkus - SN relic nu background

Ejzak - origin of the matter/antimatter asymmetry

McFarlane - strange quarks in protons/neutron charge distribution

Mengin - solar neutrino problem

Severin - reaccelerated beams

Dally - dark matter

Binning - quark-gluon plasma

Littlejohn - underground physics
Course Projects - Reading List

By October 17

Establish a reading list of at least three readable references on the subject that is finally assigned to you, and go over the reading list with me so we agree on what you will read and research for the project.
Particle Properties

- Interactions
- Flavors & States
- Charge
- Spin/Helicity
- Particle Type
- Mass
- Oscillations
\[ N \rightarrow N' + e^- \quad \text{some nuclei emit electrons!} \]

\[ M_{\text{parent}}c^2 \Rightarrow E_{\text{daughter}} + E_{\text{electron}} \]

\[ KE_{\text{electron}} = M_{\text{parent}}c^2 - M_{\text{daughter}}c^2 - m_{\text{electron}}c^2 \]

**Fig. 5.** Energy distribution curve of the beta-rays.
Pauli proposed that an undetectable particle shared the energy of beta decay with the emitted electron.
can be represented by a nuclear equation. For example,

\[ ^{238}_{92}U \rightarrow ^{4}_{2}He + ^{234}_{90}Th \]
\[ \text{α-particle} \]

In balancing nuclear equations, notice that both the total charge and the total mass must balance.

The thorium produced in the decay of \( ^{238}_{92}U \) is itself radioactive and decays by beta emission. The nuclear equation for the change is

\[ ^{234}_{90}Th \rightarrow ^{0}_{-1}e + ^{234}_{91}Pa \]
\[ \text{β-particle} \]

Thus beta emission increases the atomic number by one unit, but has (essentially) no effect on the mass.

Gamma radiation, as we learned in Chapter 4, is really nothing more than a very energetic form of electromagnetic radiation. Its emission from a nucleus doesn’t change the charge or mass number, so gamma radiation is often omitted from nuclear equations.
Fermi’s Theory of Beta Decay based on Pauli’s Letter of Regrets

Experiment: \[ M_n c^2 \neq E_p + E_e \]

Conjecture: \[ M_n c^2 = E_p + E_e + E_\nu \]

Consistency requires that \( E_\nu \) is not observable!

Fermi’s theory still stands (parity violation added in the 50s).
Fermi’s Theory

Fermi proposed electromagnetic analog

electromagnetic analog

current-current but no counterpart to electric field
Weak Interactions in the Standard Model

The weak gauge bosons $W^\pm$ act on left-handed doublets (charged-current interaction)

$\beta$-decay

Since $m_w = 80.4$ GeV $\gg m_p$, decay is governed by Fermi coupling $G_F$

Fermi coupling $\frac{G_F}{\sqrt{2}} = \frac{g_2^2}{8m_w^2}$

$g_2$ = $W$ gauge coupling

Weinberg angle $\frac{e}{g_2} = \sin \theta_W = 0.48$
Fermi’s Idea for Measuring $m_\nu$

Fig. 5. Energy distribution curve of the beta-rays.

Fig. 1.2. Graph from Fermi's famous paper on the theory of beta decay, showing how the shape of the emitted electron's energy spectrum varies with neutrino mass.
First Direct Detection of the (Anti)Neutrino

Reines and Cowan 1956

$$\bar{\nu}_e + p \rightarrow e^+ + n$$
Discovery of Muon Neutrino

1962

$\bar{\nu}_\mu + p \rightarrow n + \mu^+$

$\nu_\mu + n \rightarrow p + \mu^-$

Lederman, Schwarts, Steinberger

µ produce nice tracks as they go through the chamber (29 events)

e produce showers as they cross Al (0 events)
Muon Neutrino Mass Studies

Current best limit from studies of the kinematics of $\pi \rightarrow \mu \nu$ decay

$$p_{\mu}^2 + m_{\mu}^2 = \left( m_{\pi}^2 + m_{\mu}^2 - m_{\nu}^2 \right)^2 / 4 m_{\pi}^2$$

Can use $\pi$-decay:

**At Rest:** Mass of $\pi$ is dominant uncertainty

**In Flight:** Resolution on $p_{\pi} - p_{\mu}$ limited experimentally

Best mass limit is from $\pi$-decay at rest

$< 170$ keV at 95% CL \hspace{1cm} (Assamagan et al., PRD 1996)

New BNL Experiment using $g$-$2$ setup (sensitivity for $> 8$ keV)
Proposed BNL “NuMass” Experiment

**BNL g-2 Neutrino Mass Experiment**

\[ m(\nu_\mu) < 8 \text{ keV/c}^2 \]

Forward-going decay muons orbit a larger diameter by \( \Delta D \)

\[
\begin{align*}
\text{CM} \\
\nu_\mu & \quad \pi & \quad \mu \\
q &= 29.7 \text{ MeV/c}
\end{align*}
\]

\[
\frac{\Delta D}{D} = \frac{p_\mu - p_\pi}{p_\pi} = \frac{0.7 \text{ MeV/c}}{3 \text{ GeV/c}} = \frac{3.26 \text{ mm}}{14 \text{ m}}
\]

non-zero \( m_\nu \) shrinks \( \Delta D \)

\[
\frac{\delta D}{D} = \frac{-m_\nu^2}{2 q m_\pi}
\]

0.04 mm for current limit
Number of Active Neutrinos

Precision studies of \textit{Z-line shape}, determine number of \textit{active} light neutrinos

Each separate \((\nu_l)_L\) adds to total \textit{Z-width}.

\[
N_\nu = \frac{\frac{\Gamma_{\text{inv}}}{\Gamma_\ell}}{\frac{\Gamma_\ell}{\Gamma_\nu}} \text{SM}
\]

From LEP, one finds:

\[N_\nu = 2.984 \pm 0.008\]

which argues strongly for only having \textit{3 generations}.

Big bang nucleosynthesis gives a constraint on the effective number of light neutrinos at \(T \sim 1 \text{ MeV}\):

\[1.2 < N_\nu^\text{eff} < 3.3 \quad [99\% \text{ CL}]\]

Because mixing effects are likely to bring \textit{sterile neutrinos} into equilibrium, above suggests that the number of \((\nu_l)_R\) is also limited to \(3\).
Search for tau Neutrino

Discovery of $\tau$ lepton at SLAC

$\rightarrow$ there should be a corresponding neutrino.

In 1989, indirect evidence for the existence of $\nu_{\tau}$ in measurement of Z-width
$\rightarrow$ no one had directly observed the tau neutrino.

The tau neutrino interact and form a tau that has an 18% probability of decaying to
- a muon and two neutrinos (long event)
- an electron and two neutrinos (short event)

86% of all tau decays involve only 1 charged particle (a kink) which is the particle physicists are looking for in DONUT experiment.
Discovery of tau Neutrino

2000

An 800 GeV beam of protons from the TeVatron collides with a block of tungsten

$D_s$ decay into $\tau$ and $\nu_\tau$ neutrino

$D_s \rightarrow \nu_\tau + \tau$

$\tau \rightarrow \nu_\tau + X$

Experimental Challenges:
- Very short lifetime of the $\tau$.
- $\nu_\tau$ is extremely non-interacting (detector must have a very fine resolution).

Detecting a $\tau$ Neutrino

6,000,000 candidate events on tape

4 clean tau events
A $\nu_\tau$ interacted with a nucleon in a steel layer, producing a $\tau$.

Long tau decay because it decays to one charged particle, the electron, and the decay vertex occurs several sheets downstream from the neutrino interaction vertex.

Short tau decay to an electron in less than the distance it takes to traverse an emulsion layer.
Direct $\nu_\tau$ Mass Limits

Look at tau decays near the edge of the allowed kinematic range

\[ \tau^- \rightarrow 2\pi^- \pi^+ \nu_\tau \quad \text{and} \quad \tau^- \rightarrow 3\pi^- 2\pi^+ (\pi^0) \nu_\tau \]

Fit to scaled visible energy vs. scaled invariant mass
(e.g. hep-ex/9906015, CLEO)

Massive $\nu_\tau$ shifts edge of the kinematic distribution

Outer lines: \( m_{\nu_\tau} = 0 \)
Inner lines: \( m_{\nu_\tau} = 30 \text{ MeV} \)

Best limit is \( m(\nu_\tau) < 18.2 \text{ MeV} \) at 95% CL (Aleph, EPJ C2 395 1998)
“Standard Model” Neutrino Physics

1914  Electron Spectrum in $\beta$ decay is continuous

1930  Pauli postulates that a new particle is emitted

1933  Fermi names the new particle neutrino and introduces four-fermion interaction

1956  Reines and Cowan discover the neutrino

1962  At least two neutrinos: $\nu_e \neq \nu_\mu$

1973  Discovery of neutral currents at CERN

1983  Discovery of the W and Z

1989  Measurement of Z width at CERN $\to N_\nu=3$

2002  tau neutrino discovered.
Elementary Particles

3 $\nu$ flavors

Only upper limits on $m_\nu$ from kinematic studies.

Are $\nu$ massless?  
(ad hoc assumption in SM)
Neutrinos in the Standard Model

A neutrino is a neutral cousin of the electron and the other charged leptons.

Only weak interactions — carried by very heavy $W$, $Z$ particles with short ranges.

In the Standard Model, $m_\nu \equiv 0$. (The current limit on the sum of the three masses is $\sim 0.6$ eV). Neutrinos are many orders of magnitude lighter than the other fermions.
Different Kinds of Neutrino Flavors

Each charged lepton ($e$, $\mu$, $\tau$) has its own kind of neutrino. For example, in these reactions you get:

\[ p + e^- \rightarrow \nu_e + n \]
\[ p + \mu^- \rightarrow \nu_\mu + n \]

Note that the number of particles of each flavour type seems to be conserved in each reaction.

Flavour is also conserved in the other direction:

\[ \nu_e + n \rightarrow p + e^- \]
\[ \nu_\mu + n \rightarrow p + \mu^- \]

In the Standard Model lepton flavour is rigorously conserved, but is not protected by any symmetry of the Lagrangian.
Experimental Indications for Neutrino Oscillations

**LSND Experiment**
- \( L = 30 \text{m} \)
- \( E = \sim 40 \text{ MeV} \)

\[ \bar{\nu}_\mu \text{ } \circledR \text{ } \bar{\nu}_e \]

**Atmospheric Neutrinos**
- \( L = 15 \text{ - } 15,000 \text{ km} \)
- \( E = 300 \text{ - } 2000 \text{ MeV} \)

\[ \nu_\mu \text{ } \circledR \text{ } \nu_x \]

\( \nu_\mu \sim 66\% \)
\( \nu_e \sim 33\% \)

\( \Delta m^2 = \sim 2 \text{ to } 8 \times 10^{-5} \text{ eV}^2 \)
\( \text{Prob}_{OSC} = \sim 100\% \)

**Solar Neutrinos**
- \( L = 10^8 \text{ km} \)
- \( E = 0.3 \text{ to } 10 \text{ MeV} \)

\[ \nu_e \text{ } \circledR \text{ } \nu_x \]

\( \Delta m^2 = \sim 0.3 \text{ to } 3 \text{ eV}^2 \)
\( \text{Prob}_{OSC} = 0.3 \% \)

\( \Delta m^2 = \sim 1 \text{ to } 7 \times 10^{-3} \text{ eV}^2 \)
\( \text{Prob}_{OSC} = \sim 100\% \)
Neutrino Oscillation

Neutrino States

![Mass States and Weak States Diagram]

\[ |\nu_a\rangle = \cos \theta |\nu_1\rangle - \sin \theta |\nu_2\rangle \]
\[ |\nu_b\rangle = \sin \theta |\nu_1\rangle + \cos \theta |\nu_2\rangle \]

\[ \begin{pmatrix} v_e \\ v_\mu \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ 2\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \]

Time Evolution

\[ P_{i\rightarrow i} = \sin^2 2\theta \sin^2 \left( 1.27\Delta m^2 \frac{L}{E} \right) \]

oscillation → energy and baseline-dependent effect

Pontecorvo, 1968
Discovery of Massive Neutrinos through Oscillations

- Neutrinos are not massless
- Evidence for neutrino flavor conversion $\nu_e \leftrightarrow \nu_\mu \leftrightarrow \nu_\tau$
- Experimental results show that *neutrinos oscillate*
The LSND Experiment

- 800 MeV proton beam from LANSCE accelerator
- Water target
- Copper beamstop

\[ \pi^+ \ \overset{\text{C}}{\rightarrow} \ \mu^+ \nu_\mu \rightarrow e^+ \nu_e \bar{\nu}_\mu \]

Oscillations? \rightarrow \bar{\nu}_e

LSND took data from 1993-98
- 49,000 Coulombs of protons
- \( L = 30 \text{m} \) and \( 20 < E_\nu < 53 \text{ MeV} \)

Saw an excess of:
\[ 87.9 \pm 22.4 \pm 6.0 \text{ events.} \]

With an oscillation probability of:
\[ (0.264 \pm 0.067 \pm 0.045)\%. \]

3.8 \( \sigma \) evidence for oscillation.
Other oscillations? Sterile Neutrinos?

$\nu_{\mu} \Rightarrow \nu_e$ ?

$\nu_{\mu} \Rightarrow \nu_\tau$

$\nu_e \Rightarrow \nu_{\mu,\tau}$

Cannot be explained by 3 active neutrinos!

Will be checked by MiniBoone at FNAL (2005)
Quark Flavor Mixing

In reality, W particle couplings mix quark generations:

\[
\begin{pmatrix}
  u \\
  d'
\end{pmatrix}
\begin{pmatrix}
  c \\
  s'
\end{pmatrix}
\begin{pmatrix}
  t \\
  b'
\end{pmatrix}
\]

We say that flavour eigenstates (eg. d,s,b) are *rotated* with respect to weak eigenstates (d’,s’,b’)

\[
\begin{pmatrix}
  d' \\
  s' \\
  b'
\end{pmatrix} =
\begin{bmatrix}
  V_{ud} & V_{us} & V_{ub} \\
  V_{cd} & V_{cs} & V_{cb} \\
  V_{td} & V_{ts} & V_{tb}
\end{bmatrix}
\begin{pmatrix}
  d \\
  s \\
  b
\end{pmatrix}
\]

This allows generation-mixing decays such as \( \Lambda(uds) \rightarrow p\pi^- \)
Neutrino Mixing

Since $\nu$'s have only weak interactions, flavour eigenstates are defined as those states that couple to $W$

What if the flavour eigenstates are rotated relative to the mass eigenstates (eigenstates of Hamiltonian with well-defined mass)?

\[
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix} =
\begin{bmatrix}
U_{e1} & U_{e2} & U_{e3} \\
U_{\mu1} & U_{\mu2} & U_{\mu3} \\
U_{\tau1} & U_{\tau3} & U_{\tau3}
\end{bmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix}
\]
Evolution of Mass Eigenstates in Vacuum

\[ |\nu_e\rangle = \cos \theta |\nu_1\rangle + \sin \theta |\nu_2\rangle \]
\[ |\nu_\mu\rangle = -\sin \theta |\nu_1\rangle + \cos \theta |\nu_2\rangle \]

Each term evolves with a phase factor of \( e^{i(px-Et)} \)

If \( m_1 \neq m_2 \), then arguments of exponential will be different! For example, if we consider \( p \) to be fixed, then
\[ E = \sqrt{p^2 + m^2} = p\sqrt{1 + m^2/p^2} \approx p + m^2/(2p) \]

As neutrino propagates, a phase difference develops between terms!
\[ |\nu(t)\rangle \propto \cos \theta |\nu_1\rangle + e^{i\phi} \sin \theta |\nu_2\rangle \]

with
\[ \phi = \left( \frac{m_1^2}{2p} - \frac{m_2^2}{2p} \right) t \]
Neutrino Oscillation

Net result: at some later time, $|\nu(t)\rangle \neq |\nu_e\rangle$.

Probability that the original $\nu_e$ is detected as a $\nu_\mu$ at some later time:

$$P(\nu_e \rightarrow \nu_\mu) = |\langle \nu_\mu | \nu(t) \rangle|^2 = \sin^2 2\theta \sin^2 \left( \frac{1.27 \Delta m^2 L}{E} \right)$$

$\theta$ = neutrino mixing angle

$\Delta m^2$ = $m_1^2 - m_2^2$ (in eV$^2$)

$L$ = distance $\nu$ has travelled (in km)

$E$ = neutrino energy (in GeV)

Neutrino oscillation:

- requires at least one non-zero neutrino mass
- requires non-zero mixing elements
- results from the QM of the propagation, *not* from an interaction
Matter Effects in Neutrino Oscillation

Surprisingly the oscillation formula can be dramatically altered in matter!

\[ i \frac{d}{dt} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} -\frac{\Delta m^2}{4E} \cos 2\theta + \sqrt{2} G_F N_e & \frac{\Delta m^2}{4E} \sin 2\theta \\ \frac{\Delta m^2}{4E} \sin 2\theta & \frac{\Delta m^2}{4E} \cos 2\theta \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} \]

The relevant process is forward scattering, in which no momentum is exchanged. In matter, \( \nu_e \)'s have a different forward scattering amplitude than the other flavours:

<table>
<thead>
<tr>
<th>At Solar Neutrino Energies:</th>
<th>( \nu_x )</th>
<th>( \nu_x )</th>
<th>( \nu_e )</th>
<th>e(^-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>e(^-)</td>
<td>Z(^0)</td>
<td>W(^+)</td>
<td>e(^-)</td>
<td>( \nu_e )</td>
</tr>
</tbody>
</table>

All neutrino flavors Only electron neutrinos

The size of the potential is proportional to the electron density \( N_e \).

For solar \( \nu \)'s, matter effects are dominant.

This produces a matter-induced potential that is different for \( \nu_e \). Effectively \( \nu_e \)'s have a different “index of refraction” in matter.
Neutrino Mixing Matrix

\[ U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \]

Adjust \( L/E \) to view oscillations at different \( \Delta m^2 \)’s

Atmospheric \( \nu \)’s:
\[ \theta_{23} \approx \pi/4 \]
Maximal mixing! (?)

Short baseline reactor \( \nu \)’s:
\[ \theta_{13} < \pi/20 \]
Small, quark-like mixing

Solar \( \nu \)’s:
\[ \theta_{12} \approx \pi/6 \]
Large, non-maximal mixing

Compare to identical parameterization of CKM matrix ...

\[ \theta_{23} \approx \pi/76 \]
\[ \theta_{13} \approx \pi/870 \]
\[ \theta_{12} \approx \pi/14 \]
Atmospheric Neutrinos

Incident proton strikes atmosphere, making pion

\[ \pi \rightarrow \mu + \nu_{\mu} \]
\[ \rightarrow e + \nu_{\mu} + \nu_{e} \]

Two muon neutrinos produced for each electron neutrino!

SuperKamiokande detector
Neutrino Mixing Matrix/Leptonic Unitarity Triangle

Global Fit of Oscillation Data ($3\sigma$)

$$U_{PMNS} = \begin{pmatrix}
U_{e1} & U_{e2} & U_{e3} \\
U_{\mu1} & U_{\mu2} & U_{\mu3} \\
U_{\tau1} & U_{\tau2} & U_{\tau3}
\end{pmatrix} = \begin{pmatrix}
0.78 - 0.88 & 0.47 - 0.62 & 0.0 - 0.23 \\
0.18 - 0.55 & 0.40 - 0.73 & 0.57 - 0.82 \\
0.19 - 0.55 & 0.41 - 0.75 & 0.55 - 0.82
\end{pmatrix}$$

Can we reconstruct the triangle? Can we use it to determine the CP-violating phase?

|U_{e3}| = 0.16

nearly best fit values of other angles

$$J = s_{12} c_{12} s_{13} c_{13}^2 s_{23} c_{23} \sin \delta$$

Problem: coherence (we deal with coherent states and not mass eigenstates of neutrinos)
# Quarks and Leptons

## Mixing

<table>
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<tr>
<th></th>
<th>Quarks</th>
<th>Leptons</th>
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<tr>
<td>1-2, $\theta_{12}$</td>
<td>13°</td>
<td>32°</td>
</tr>
<tr>
<td>2-3, $\theta_{23}$</td>
<td>2.3°</td>
<td>45°</td>
</tr>
<tr>
<td>1-3, $\theta_{13}$</td>
<td>~ 0.5°</td>
<td>&lt;13°</td>
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$$\theta_{12} + \theta_C = \theta_{23} \sim 45°$$

## Hierarchy of Mass

### Neutrinos
- $|m_2/m_3| > 0.18$

### Charged leptons
- $|m_\mu/m_\tau| = 0.06$

### Down quarks
- $|m_s/m_b| \sim 0.02 - 0.03$

### Up-quarks
- $|m_c/m_t| \sim 0.005$

Ref: Smirnov
Too Many $\nu$-Oscillation Signals

Three known neutrinos $\nu_e \nu_\mu \nu_\tau$ cannot explain three different $\Delta m^2$ values.

\[ \Delta m_{\text{solar}}^2 = m_2^2 - m_1^2 = 5 \times 10^{-5} \text{eV}^2 \]
\[ \Delta m_{\text{atmos}}^2 = m_3^2 - m_2^2 = 3 \times 10^{-3} \text{eV}^2 \]

then
\[ \Delta m_{\text{LSND}}^2 = m_3^2 - m_1^2 = \sim 5 \times 10^{-3} \text{eV}^2 \]

But LSND sees $\sim 1 \text{eV}$

Experimental ideas
- Not all 3 signals are neutrino oscillations
- Unknown uncertainties give false signals

Theoretical ideas
- Neutrinos and antineutrinos have different masses
- More than three types of neutrinos – extra “sterile” neutrino types

with $\Delta m^2 > 0.1 \text{eV}^2$

$m(\nu)$ in cosmologically relevant region?