Physics 736

Experimental Methods in Nuclear-, Particle-, and Astrophysics

- Passage of Particles & Radiation Through Matter -

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Know your interaction....

• What are you measuring/analyzing in your research?
  – what is the signal?
  – what interactions? energy spectrum?
  – what is the particle?
  – what interactions take place?
  – what is the cross-section?

→ HW#2
Interaction of Charged Particles

- charged particles
  - inelastic collisions w/ atomic e-
  - elastic scattering
  - Cherenkov radiation
  - energy loss of electrons and positrons
    - collisions
    - Brehmsstrahlung
  - Coulomb scattering
  - nuclear reactions
  - energy loss distributions
Interaction of Charged Particles

• characteristic features
  – energy loss
  – deflection of particles from incident direction

• classes of particles
  – $e^+, e^-$
  – heavy particles: $\mu, \pi, p, \alpha$

• primary processes
  – inelastic collisions
  – elastic scattering
Review of Last Lecture

What is stopping power?

\[ S(E) = -\frac{dE}{dx} \]
Review of Last Lecture

Stopping power (MeV/(g/cm²))

\[ \beta_\gamma \]
Review of Last Lecture

The diagram shows the relationship between stopping power (in MeV/(g/cm$^2$)) and $\beta\gamma$.

The stopping power peaks at a certain value of $\beta\gamma$ before decreasing and then increasing again as $\beta\gamma$ increases further.
Review of Last Lecture

Which region is described by the Bethe-Bloch formula?
Which region is described by the Bethe-Bloch formula?
Stopping Power

Bohr (classical treatment)

\[- \frac{dE}{dx} = \frac{4\pi z^2 e^4}{m_e v^2} N_e \ln \frac{\gamma^2 m v^3}{z e^2 \nu} \]

Bethe-Bloch (QM)

\[- \frac{dE}{dx} = 2\pi N_a \frac{r_e^2 m_e c^2 \rho Z}{A} \frac{Z^2}{\beta^2} \left[ \ln \left( \frac{2 m_e \gamma^2 v^2 W_{\text{max}}}{I^2} \right) - 2 \beta^2 \right] \]

\[I = \text{mean excitation potential}\]
Stopping Power

Bethe-Bloch

The figure shows the stopping power $dE/dx$ as a function of energy $E$ for two cases: with and without corrections. The graph is plotted on a log-log scale, with $dE/dx$ in MeV cm$^{-2}$/g and energy $E$ in MeV.

For high-energy particles, the stopping power decreases rapidly with energy, while for low-energy particles, it increases more gradually. The dashed line represents the case with corrections, while the solid line shows the case without corrections. The corrections are significant at higher energies but become less important at lower energies.
Review of Last Lecture

• What is the approximate speed of minimum ionizing particles?
Stopping Power

Energy Dependence of Bethe-Bloch

non-relativistic $E$

\[ \frac{dE}{dx} \propto \frac{1}{\beta^2} \]

each particle has different $dE/dx$.

minimum ionizing

\[ v = 0.96c \]

same point for all particles of same charge

high $E$

most relativistic particles have energy loss rates close to the minimum

+ radiation losses for $e^{+/-}$
Stopping Power

Bethe-Bloch

\[-\frac{dE}{dx} = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A} \frac{z^2}{\beta^2} \left[ \ln \left( \frac{2m_e \gamma^2 v^2 W_{\text{max}}}{I^2} \right) - 2 \beta^2 \right] \]

with density and shell corrections

\[-\frac{dE}{dx} = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A} \frac{z^2}{\beta^2} \left[ \ln \left( \frac{2m_e \gamma^2 v^2 W_{\text{max}}}{I^2} \right) - 2 \beta^2 - \delta - 2 \frac{C}{Z} \right] \]

typically \( dE/dx \) depends only on \( \beta \) (given a particle and medium)
Stopping Power

Stopping power for muons in copper

\[ S(E) = -\frac{dE}{dx} \]

- nuclear inelastic collisions if you consider nuclei instead of muons
- bremsstrahlung important for electrons and muons

stopping power (MeV/(g/cm²))

Bethe–Bloch
Bremsstrahlung

minimum ionizing particles

stopping power less because ions attach electrons
Stopping Power

Bethe-Bloch

At low $\beta$, $-dE/dx \propto 1/\beta^2$ decreases rapidly as $\beta$ increases. Reaches a min at $\beta\gamma \approx 3$ (a particle at the energy loss min is called mip).

Typically $dE/dx$ depends only on $\beta$ (given a particle and medium).
Stopping Power

Bethe-Bloch

low momentum region where \(-dE/dx \propto 1/\beta^2\) and the relativistic rise depend on \(m\) so can be used for particle identification (PID)
Stopping Power

Bethe-Bloch

For a given particle (z) and target (I,N,Z,A), the energy loss depends only on the velocity of the particle!

Most relativistic particles have energy loss rates close to the minimum (mip = minimum ionizing particles)~ 2 MeV/g/cm^2
Stopping Power

• if we know dE/dx for one particle we can scale to another one!

• dE/dx varies little when expressed in mass thickness
Stopping Power

**scaling laws**

if we know $\frac{dE}{dx}$ for one particle we can scale to another one

$$\frac{dE_2}{dx}(T_2) = -\frac{Z_2^2}{Z_1^2} \frac{dE_1}{dx} \left( T_2 \frac{M_1}{M_2} \right)$$

**mass thickness**

dE/dx varies little when expressed in terms of mass thickness
Stopping Power

stopping power for muons in copper

[Graph showing stopping power for muons in copper with various loss processes and momentum scales.]
Stopping Power

**electronic and nuclear stopping**

**electronic stopping =**
- slowing down due to the inelastic collisions between bound electrons in the medium and the ion moving through it
- collisions may result in excitations

**nuclear stopping =**
- elastic collisions between the ion and atoms in the sample

**Resources**
The stopping and range of ions in matter
http://www.SRIM.org/

Stopping Power for light ions
http://www.exphys.uni-linz.ac.at/Stopping/
Stopping Power

electronic and nuclear stopping

![Graph showing stopping power vs. particle energy]
Example: cosmic muons and plastic scintillator
Sto\-pping Power

Bragg Curve

dE/dx depends on kinetic energy

charged particle is more ionizing towards the end of its path

heavy particles pick up electrons towards the end
energy loss follows statistical distribution

range straggling (=statistical distribution of energy losses)

energy loss distribution
Range

Figure 27.4: Range of heavy charged particles in liquid (bubble chamber) hydrogen, helium gas, carbon, iron, and lead. For example: For a $K^+$ whose momentum is 700 MeV/c, $\beta\gamma = 1.42$. For lead we read $R/M \approx 396$, and so the range is 195 g cm$^{-2}$.

In older references [4,5] the "low-energy" approximation $T_{\text{max}} = 2m_e c^2 \beta^2 \gamma^2$, valid for $m_e / M \ll 1$, is often implicit. For a pion in copper, the error thus introduced into $dE/dx$ is greater than 6% at 100 GeV.

At energies of order 100 GeV, the maximum 4-momentum transfer to the electron can...
Channeling

Critical angle for channeling

\[ \phi_c = \frac{\sqrt{zZA_0A_d}}{1670\beta\sqrt{\gamma}} \]

channeling = reduced energy loss
Cherenkov Radiation

\[ \cos \theta = \frac{c}{v n} \]

\( v = \text{particle velocity} \)
\( n = \text{index of refraction of the medium} \)

For water with \( n = 1.33 \), the limiting angle for high speed particles is given by:

\[ \theta = \cos^{-1} \left( \frac{1}{1.33} \right) = 41.2^\circ \]

The threshold particle speed for Cherenkov radiation is \( v = c/n \), which for an electron in water gives a threshold particle kinetic energy of 0.26 MeV.

\[ \beta = 0.752, \quad E_{\text{electron}} = \gamma m_e c^2 = \frac{1}{\sqrt{1 - \beta^2}} m_e c^2 = (1.52)(0.511 \text{ MeV}) = 0.775 \text{ MeV} \]

Kinetic energy = 0.775 MeV - 0.511 MeV = 0.26 MeV

\[ v_{\text{particle}} > \frac{c}{n} \]

\[ \cos \theta_c = \frac{1}{\beta n(w)} \]
Cherenkov Radiation

The Cerenkov radiation from a muon produced by a muon neutrino event yields a well defined circular ring in the photomultiplier detector bank.

The Cerenkov radiation from the electron shower produced by an electron neutrino event produces multiple cones and therefore a diffuse ring in the detector array.
Cherenkov Radiation

Sudbury Neutrino Observatory

electron neutrino $\nu_e$

Cerenkov Light

electron $e^-$

protons

Deuteron

neutrino $\nu$

electron $e^-$

neutrino $\nu$

Sudbury Neutrino Observatory
Cherenkov Counters

Longitudinal view of the ALICE detector.